OAK HALL SCHOOL

2024-2025

Suggested Review Exercises
for students entering

AP Calculus AB

A message from the Math Department

Mathematics is a subject that is cumulative in nature as it constructs new knowledge from foundational prior knowledge. Therefore, as it is imperative to our students' success, we require them to have mastered certain skills and concepts before entering a new math course.

Each course in the math department has provided suggested exercises for incoming students as a resource for them to review the required prerequisites that are critical to their success in the course. While we will not be requiring students to complete these exercises as a formal assignment to be turned in, we have the highest expectations of our students as self-aware, proactive learners. Each student is responsible for gauging which prerequisites they need to reinforce and how much studying they need to do for them to start the new school year feeling confident, prepared, and accomplished.

We recommend that our students begin this process mid to late summer in order for everything to be fresh in their minds but also to give them time to recover from the school year they just completed. Rest is not an indulgence; it is a human necessity. We hope everyone has a safe, fun, and restful summer and we look forward to having another great school year when we come back in August!

1) Simplify: (a) $\dfrac{x^3-9x}{x^2-7x+12}$ (b) $\dfrac{x^2-2x-8}{x^3+x^2-2x}$

2) Rationalize the denominator and simplify: $\dfrac{x-4}{\sqrt{x}-2}$

3) Solve for y: $\ln(y-1)=\dfrac{x^2}{8}+\ln 2-\dfrac{1}{8}$

4) Simplify: (a) $\log_2 5+\log_2(x^2-1)-\log_2(x-1)$ (b) $2\log\sqrt{x}+3\log x^{1/3}$

5) Expand: (a) $\ln\left(\dfrac{3x-1}{(3x+1)^2}\right)^5$ (b) $\log\left(4x^3\sqrt{x^2-16}\right)$

6) Solve the following equations for the indicated variables:

(a) $A=2\pi r^2+2\pi rh$, for positive r (b) $2x-2yd=y+xd$, for d (c) $\dfrac{2x}{4\pi}+\dfrac{1-x}{2}=0$; for x

7) Factor completely: (a) x^6-16x^4 (b) $4x^3-8x^2-25x+50$ (c) $8x^3+27$ (d) x^4-1

8) Find <u>all</u> real solutions to: (a) $x^6-16x^4=0$ (b) $4x^3-8x^2-25x+50=0$ (c) $8x^3+27=0$

9) Without the use of a calculator, evaluate the following:

(a) $\cos\dfrac{7\pi}{6}$ (b) $\sin\dfrac{5\pi}{4}$ (c) $\tan^{-1}(-1)$ (d) $\sin^{-1}(-1)$

(e) $\cos\dfrac{9\pi}{4}$ (f) $\sin^{-1}\dfrac{\sqrt{3}}{2}$ (g) $\tan\dfrac{7\pi}{6}$ (h) $\cos^{-1}(-1)$

10) Solve for x: (a) $3\sin^2 x=\cos^2 x;\ 0\le x<2\pi$ (b) $\cos^2 x-\sin^2 x=\sin x;\ -\pi<x\le\pi$

(c) $\tan x+\sec x=2\cos x;\ -\infty<x<\infty$

11) Use long division to find $\left(x^5-x^4+x^3+2x^2-x+4\right)\div\left(x^3+1\right)$.

12) Solve for x: (a) $|5x-2|=8$ (b) $|2x+1|=x+3$

13) Determine the equations of the following lines in point-slope and slope intercept form:

(a) the line through $(-1,3)$ and $(2,-4)$ (b) the line through $(-1,2)$ and perpendicular to the line $2x-3y+5=0$

14) Find the point of intersection of the lines: $3x-y-7=0$ and $x+5y+3=0$

15) Find the equation of the circle that passes through the origin and has intercepts equal to 1 and 2 on the x-axis and y-axis, respectively.

16) A curve is traced by a point $P(x,y)$, which moves such that its distance from the point $A(-1,1)$ is three times its distance from the point $B(2,-1)$. Determine the equation of the curve.

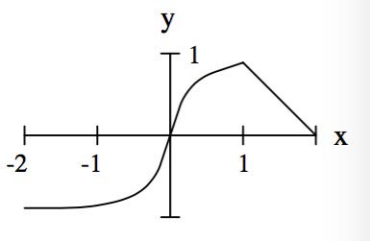
17) Let $f(x)=\frac{|x|}{x}$. Show that $f(x)=\begin{cases} 1, & x>0 \\ -1, & x<0 \end{cases}$. Find the domain and range of $f(x)$.

18) Simplify $\frac{f(x+h)-f(x)}{h}$, where (a) $f(x)=2x+3$ (b) $f(x)=\frac{1}{x+1}$ (c) $f(x)=x^2$

19) The graph of the function $y=f(x)$ is shown at the right:

Determine the graphs of the functions:

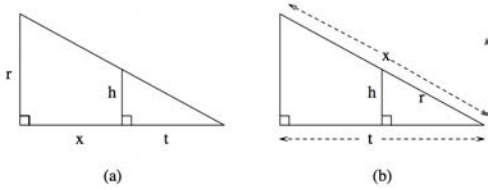
(a) $f(x+1)$ (b) $f(-x)$ (c) $|f(x)|$



20) Let $g(x)$ be a monotonic, one-to-one function shown in the table below. Find $g^{-1}(2)$.

x	-6	1	2	3	10	29
y	-2	-1	0	1	2	3

21) Express x in terms of the other variables in the picture.



22) Find the ratio of the area inside the square but outside the circle.

23) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

24) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal. Assume that the string is perfectly straight.

25) You should know the following trigonometric identities.

(A) $\sin(-x)=?$ (B) $\cos(-x)=?$ (C) $\sin^2 x+\cos^2 x=?$ (D) $\sin 2x=?$

26) Simplify: (a) $\dfrac{\frac{1}{\sin^2\theta}-1}{\frac{\cos^2\theta}{\sin^2\theta}}$ (b) $\dfrac{\frac{1}{\sin\theta}-\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\sin\theta}}$

Solutions

1) a) $\frac{x(x+3)}{x-4}$ b) $\frac{x-4}{x(x-1)}$ 2) $\sqrt{x}+2$ 3) $y=2e^{\frac{x^2-1}{8}}+1$

4) a) $\log_2 5(x+1)$ b) $2\log x$

5) a) $5\left(\ln(3x-1)-2\ln(3x+1)\right)$ b) $\log 4x^3+\frac{1}{2}\log(x+4)+\frac{1}{2}\log(x-4)$

6) a) $r=-\frac{h}{2}\pm\sqrt{\frac{A}{2\pi}+\frac{h^2}{4}}$ b) $d=\frac{2x-y}{x+2y}$ c) $x=\frac{-\pi}{1-\pi}$

7) a) $x^4(x+4)(x-4)$ b) $(2x+5)(2x-5)(x-2)$ c) $(2x+3)\left(4x^2-6x+9\right)$

d) $\left(x^2+1\right)(x+1)(x-1)$

8) a) $x=0,-4,4$ b) $x=-\frac{5}{2},\frac{5}{2},2$ c) $x=-\frac{3}{2}$

9) a) $-\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{2}}{2}$ c) $\frac{3\pi}{4},\frac{11\pi}{4}$ d) $\frac{3\pi}{2}$ e) $\frac{\sqrt{2}}{2}$ f) $\frac{\pi}{3},\frac{2\pi}{3}$ g) $\frac{\sqrt{3}}{3}$ h) π

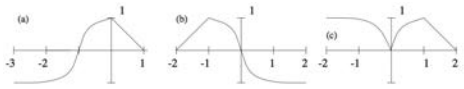
10) a) $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$ b) $x=\frac{\pi}{6},\frac{5\pi}{6},-\frac{\pi}{2}$ c) $x=\frac{\pi}{6}+2n\pi,\frac{5\pi}{6}+2n\pi$, where n is an integer

11) $x^2+\frac{x^2-x+4}{x^3+1}$ 12) a) $x=2,-\frac{6}{5}$ b) $x=2,-\frac{4}{3}$

13) a) $y-3=-\frac{7}{3}(x+1)$ or $y+4=-\frac{7}{3}(x-2)$; $y=-\frac{7}{3}x+\frac{2}{3}$

b) $y-2=-\frac{3}{2}(x+1)$; $y=-\frac{3}{2}x+\frac{1}{2}$ 14) $(2,-1)$ 15) $\left(x-\frac{1}{2}\right)^2+(y-1)^2=\frac{5}{4}$

16) $8x^2-38x+8y^2+20y+43=0$ (a circle) 17) Domain: $(-\infty,0)\cup(0,\infty)$ Range: $y=-1,1$

18) a) 2 b) $\frac{-1}{(x+1)(x+h+1)}$ c) $2x+h$ 19) 

20) 10 21) a) $x=t\left(\frac{r-h}{h}\right)$ b) $x=\frac{rt}{\sqrt{r^2-h^2}}$ 22) $1-\frac{\pi}{4}$ 23) $\frac{9\pi}{4}$ 24) $\frac{\pi}{6}$

26) a) 1 b) $-\frac{1}{\sin\theta+\cos\theta}$