OAK HALL SCHOOL

2024-2025

Suggested Review Exercises
for students entering

AP Calculus BC

A message from the
Math Department

Mathematics is a subject that is cumulative in nature as it constructs new knowledge from foundational prior knowledge. Therefore, as it is imperative to our students' success, we require them to have mastered certain skills and concepts before entering a new math course.

Each course in the math department has provided suggested exercises for incoming students as a resource for them to review the required prerequisites that are critical to their success in the course. While we will not be requiring students to complete these exercises as a formal assignment to be turned in, we have the highest expectations of our students as self-aware, proactive learners. Each student is responsible for gauging which prerequisites they need to reinforce and how much studying they need to do for them to start the new school year feeling confident, prepared, and accomplished.

We recommend that our students begin this process mid to late summer in order for everything to be fresh in their minds but also to give them time to recover from the school year they just completed. Rest is not an indulgence; it is a human necessity. We hope everyone has a safe, fun, and restful summer and we look forward to having another great school year when we come back in August!

Section I

This section contains 45 multiple-choice questions and contains two parts: Part A and Part B. Part A has 28 questions that must be solved without a calculator. Part B has 17 questions, including some questions that require the use of a graphing calculator.

Part A Multiple-Choice

You may not use a calculator on this portion of the exam.

Directions: Solve each problem in the provided space. Then choose the best option from among the choices given. Be efficient with the use of your time.

Throughout this exam:

(1) The domain of each function f is the set of all real numbers for which $f(x)$ is defined. If the domain of a particular function differs from this, it will be specified in the problem.

(2) For trigonometric functions, the inverse may be represented with "$^{-1}$" or "arc". For example, the inverse cosine function may be represented as $\cos^{-1} x$ or $\arccos x$.

1. $\int \sin\left(\frac{1}{4}x\right) dx =$

(A) $-\cos\left(\frac{1}{4}x\right)+C$

(B) $-4\cos\left(\frac{1}{4}x\right)+C$

(C) $4\cos\left(\frac{1}{4}x\right)+C$

(D) $-\frac{1}{4}\cos\left(\frac{1}{4}x\right)+C$

(E) $\frac{1}{4}\cos\left(\frac{1}{4}x\right)+C$

2. Find $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x - 1}$.

(A) 2

(B) 1

(C) 0

(D) -1

(E) Does not exist

3. For what value(s) of k, if any, is $f(x) = \begin{cases} k^2 - x^2 & \text{if } x < 2 \\ 2(x + k) & \text{if } x \geq 2 \end{cases}$ continuous on $(-\infty, \infty)$?

(A) $-4, -2$

(B) 2

(C) 4

(D) $-2, 4$

(E) Does not exist

4. If $f(x) = 2x\sqrt{8x - 1}$, then $f'(x)$ is

(A) $x\sqrt{8x - 1}$

(B) $\frac{8x}{\sqrt{8x - 1}}$

(C) $2\sqrt{8x - 1} + 8x$

(D) $2\sqrt{8x - 1} + \frac{8x}{\sqrt{8x - 1}}$

(E) $2 + \frac{8x}{\sqrt{8x - 1}}$

5. $\int \frac{8}{1+8x} dx =$

(A) $8x + \ln 8x + C$

(B) $\ln|1+8x| + C$

(C) $\ln|8x| + C$

(D) $\frac{1}{8}\ln|1+8x| + C$

(E) $-\frac{1}{8}\ln|1+8x| + C$

6. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos(4t)$. If the position of the particle at time $t = \frac{\pi}{8}$ is $x = \frac{7}{4}$, what is the particle's position at time $t = 0$?

(A) $\frac{2}{3}$

(B) $\frac{5}{4}$

(C) $-\frac{1}{2}$

(D) 0

(E) $\frac{3}{2}$

7. The function f is continuous on the closed interval [0,9] and has the values given in the table below.

x	0	3	6	9
$f(x)$	8	15	k	12

The trapezoidal approximation for $\int_0^9 f(x)dx$ found with three subintervals of equal length is 90. What is the value of k?

(A) 2

(B) 5

(C) 7

(D) 11

(E) 17

8. $\frac{d}{dx}\left(\sin^3\left(x^2\right)\right) =$

(A) $6x\cos^2\left(2x\right)$

(B) $6x\sin^2\left(x^2\right)\cos\left(x^2\right)$

(C) $2x\sin^3(x^2) + 3x^2\sin^2(2x)$

(D) $6x\sin^2\left(x^2\right)$

(E) $3\cos^2\left(2x\right)$

9. A colony of bacteria starts with 500 and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria. When will the population reach 25,000?

(A) 3.87267 hours

(B) 4.01239 hours

(C) 4.23289 hours

(D) 4.98778 hours

(E) 5.43927 hours

10. Given $f(x) = \dfrac{x^4 - 27}{x^2}$, for what values of x is the graph of f concave downwards?

(A) $-3 < x < 0$

(B) $0 < x < 3$

(C) $3 < x < \infty$

(D) $-3 < x < 0$ and $0 < x < 3$

(E) $-\infty < x < -3$ and $3 < x < \infty$

11. Let f be defined by $f(x) = |x - 6|$ for all real numbers x. For what values of x is the function increasing?

(A) $(-\infty, -6)$

(B) $(-\infty, 6)$

(C) $[-6, 0)$

(D) $(0, 6)$

(E) $(6, \infty)$

12. Find an equation for the line tangent to the graph of $f(x) = \sqrt{x - 7}$ at the point where $x = 16$

(A) $y = 6x - 2$

(B) $y = \dfrac{1}{6}x - \dfrac{1}{3}$

(C) $y = \dfrac{1}{6}x + \dfrac{1}{3}$

(D) $y = -6x - 2$

(E) $y = -\dfrac{1}{6}x + \dfrac{1}{3}$

13. $\frac{d}{dx}\left(\sin x\cos(2x)\right)=$

(A) $\cos(x)\cos(2x)-2\sin(x)\sin(2x)$

(B) $2\cos(x)\cos(2x)-\sin(x)\sin(2x)$

(C) $\cos(x)\cos(2x)+2\sin(x)\sin(2x)$

(D) $2\cos(x)\cos(2x)-2\sin(x)\sin(2x)$

(E) $2\cos(x)\cos(2x)+2\sin(x)\sin(2x)$

14. Find the function $y=f(x)$ if $\frac{dy}{dx}=2x-1$ and $f(1)=3$.

(A) x^2-x+3

(B) x^2-x+5

(C) $2x^2+x-3$

(D) $\frac{1}{2}x^2-x+\frac{5}{2}$

(E) x^2-4x+6

15. $\int_1^2 xe^{6x^2}\,dx=$

(A) $\frac{e^{12}-e^4}{8}$

(B) $\frac{e^{24}-e^6}{12}$

(C) $\frac{e^{24}-e^{12}}{12}$

(D) $\frac{e^{24}+e^6}{12}$

(E) $\frac{e^2-e^{12}}{12}$

16. How many points of inflection does $f(x) = 3x\left(x+2\right)^3$.

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

17. Find the area of the region bound by $f(x) = x^2$ and $g(x) = x^3$.

(A) $\frac{1}{6}$

(B) $\frac{1}{2}$

(C) $\frac{1}{8}$

(D) $\frac{1}{12}$

(E) $\frac{1}{16}$

18. If $F(x) = \int_1^{x^3-10} f(u)du$ and $f(-2) = 5$, then $F'(2) =$

(A) 60

(B) 30

(C) 10

(D) -5

(E) -10

19. What is the equation of the line tangent to the curve $y = x^3 - 2x^2$ at the point $(2,0)$?

(A) $y = -4x - 8$

(B) $y = 4x + 8$

(C) $y = 4x - 8$

(D) $y = \frac{1}{4}x - 8$

(E) $y = -\frac{1}{4}x + 8$

20. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 4t^3 - 4t$. Which of the following expressions could represent the position $x(t)$ of the particle at any time $t \geq 0$?

(A) $t^3 - 2t^2 + 4$

(B) $t^3 - 2t^3$

(C) $t^4 - 2t^2 + 4$

(D) $t^4 - 2t - 3$

(E) $t^4 - 4t^2 + 4$

21. What is the slope of the tangent line to $x^2 + xy + y^2 = 3$ that lies in the first quadrant at the point where $y = 1$?

(A) -2

(B) -3

(C) 3

(D) -1

(E) 1

22. If $f(x) = \ln|2x|$ then what is the derivative of the inverse of $f(x)$ at $x = 4$?

(A) $\frac{1}{2}e^2$

(B) $\frac{1}{4}e^4$

(C) $\frac{1}{4}e^2$

(D) $\frac{1}{2}e^4$

(E) Undefined

23. The function $f(x) = \dfrac{3x^4 - 5x^3 + 79}{7x^4 + 9x^2 + 11}$ has horizontal asymptote(s) at

(A) $y = \pm 3$

(B) $y = \pm\frac{7}{3}$

(C) $y = \frac{3}{7}$

(D) $y = 0$

(E) $y = 1$

24. Find the derivative of $f(x)=\dfrac{x-4}{\left(x^2+1\right)^2}$.

(A) $\dfrac{-3x^2+16x+1}{\left(x^2+1\right)^{-2}}$

(B) $\dfrac{-3x^2+16x+1}{\left(x^2+1\right)^{2}}$

(C) $\dfrac{3x^2-16x-1}{\left(x^2+1\right)^{3}}$

(D) $\dfrac{-3x^2+16x+1}{\left(x^2+1\right)^{4}}$

(E) $\dfrac{-3x^2+16x+1}{\left(x^2+1\right)^{3}}$

25. Evaluate $\dfrac{d}{dx}\int_0^x \left(t^5-7t+9\right)dt$ for $x \geq 0$.

(A) x^4-7

(B) t^5-7t

(C) $5x^4-7x$

(D) t^5-7t+9

(E) x^5-7x+9

26. Find $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}$

(A) 8

(B) 12

(C) 16

(D) 4

(E) ∞

27. Which of the following statements is true about $f(x) = -x^3 - 6x^2 - 9x - 2$?

(A) f is decreasing on $(-3, -1)$

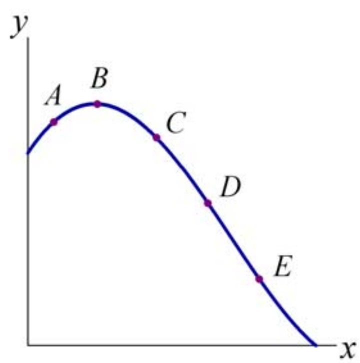
(B) f is increasing on $(-3, -1)$

(C) f is increasing on $(-\infty, -3)$

(D) f is increasing on $(-2, \infty)$

(E) f is decreasing for all real values

28. For which points on the graph of f is $f'(x)$ positive?



(A) A and E

(B) A only

(C) C only

(D) C, D, and E

(E) B only

Part B Multiple-Choice

This portion of the exam requires a graphing calculator for some questions.

This part of the exam contains 17 questions.

Directions: Solve each problem in the provided space. Then choose the best option from among the choices given. Be efficient with the use of your time.

Throughout this exam:
(1) Sometimes the exact numerical value of the solution is not given as a choice. In this event, pick the best numerical approximation from the given choices.
(2) The domain of each function f is the set of all real numbers for which $f(x)$ is defined. If the domain of a particular function differs from this, it will be specified in the problem.
(3) For trigonometric functions, the inverse may be represented with "$^{-1}$" or "arc". For example, the inverse cosine function may be represented as $\cos^{-1} x$ or $\arccos x$.

29. Find the position function, $P(t)$, given $a(t)=8$, $v(2)=9$, and $P(1)=0$ where the acceleration is given by $a(t)$ and the velocity is given by $v(t)$.

(A) $P(t)=4t^2-7t-3$

(B) $P(t)=4t^2-7t+3$

(C) $P(t)=t^2-7t+3$

(D) $P(t)=4t^2+7t+3$

(E) $P(t)=t^2+7t+3$

30. Give a value of c that satisfies the conclusion of the Mean Value Theorem for Derivatives for the function $f(x)=-2x^2-x+2$ on the interval $[1,3]$.

(A) $\frac{9}{4}$

(B) $\frac{3}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) $\frac{5}{4}$

31. The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. At the time when the volume and radius are changing at the same numerical amount, what is the volume?

(A) 4.18879

(B) 0.282095

(C) 0.094032

(D) 0.023508

(E) 0.376126

32. Given $f(x) = (1+5x)^{\frac{1}{3x}}$ find $\lim\limits_{x \to 0} f(x)$.

(A) 1.66667

(B) 1.82212

(C) 5.29449

(D) 1.06894

(E) 0.000000305902

33. Find the value of c on the interval $[0, \pi]$ that satisfies the Mean Value Theorem for Derivatives given $f(x) = x - \cos x$.

(A) 1.6366

(B) 0.6901

(C) 0.6366

(D) 1.9471

(E) 3.1415

34. If $f'(x) = 2x\sin(2x)$ for $0 < x < 4$ then f has a local max at $x \approx$

(A) 3.990

(B) 3.142

(C) 1.782

(D) 6.345

(E) 1.571

35. A rock is thrown upwards (vertically) from the ground with an initial velocity of 40 feet per second. If the acceleration due to gravity is $-10\dfrac{ft}{\sec^2}$, how high will the rock go?

(A) 20 feet

(B) 40 feet

(C) 80 feet

(D) 100 feet

(E) 120 feet

36. What is the average value of the function $f(x) = x^3 + 4x^2 - 5x + 7$ on the interval $[2,6]$?

(A) 545.333

(B) 272.667

(C) 181.778

(D) 155.809

(E) 136.333

37. An object moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 4t^3 - 4t$. Find the total displacement of the particle from $t = 0$ to $t = 2$.

(A) 0

(B) 8

(C) 9

(D) 10

(E) 12

38. Estimate the area under the curve of $f(x) = \cos(x) + 1$ from $x = 1$ to $x = 4$. Use three sub-intervals and right endpoints.

(A) 1.402

(B) 2.942

(C) 1.030

(D) 0.940

(E) 2.596

39. The base of a solid is the region in the *xy*-plane enclosed by the curves $f(x) = \sin(x), g(x) = \cos(x)$ over the interval $\left[0, \frac{\pi}{4}\right]$. Cross sections of the solid perpendicular to the *x*-axis are squares. Determine the volume of the solid.

(A) 0.2854

(B) 0.3123

(C) 0.2758

(D) 0.3085

(E) 0.3171

40. The first derivative of the function f is given by $f'(x) = 4x^3 - 18x^2$. How many points of inflection does the graph of f have?

(A) one

(B) two

(C) three

(D) four

(E) zero

41. Find the average value of $f(x) = \sin x$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

(A) 0.900316

(B) 0.707107

(C) 0.31831

(D) 0.450158

(E) −0.777594

42. If $f(x) = x^2 e^{-4x}$, then the y-value of one of the horizontal tangents is

(A) 0.5

(B) 7.38906

(C) 0.541341

(D) 0.033834

(E) 0.045112

43. Find the interval(s) on which the curve $f(x) = -x^4 + 2x^2 + 3$ is concave up.

(A) $\left(-\frac{\sqrt{3}}{3}, 0\right)$

(B) $\left(0, \frac{\sqrt{3}}{3}\right)$

(C) $\left(-\frac{3}{4}, \frac{3}{4}\right)$

(D) $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

(E) $\left(-\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{4}\right)$

44. The velocity of a particle moving along a line is $2t$ meters per second. Find the distance traveled in meters during the time interval $1 \le t \le 3$.

(A) 9

(B) 5

(C) 6

(D) 4

(E) 8

45. If $F(x) = \int_{x^2-1}^{0} f(t)dt$ and $f(8) = -4$ then $F'(3) =$

(A) 24

(B) -24

(C) 12

(D) -12

(E) 32

Section II

This section contains six free response problems and has two parts: Part A and Part B. Part A has three questions that may require a calculator. A calculator may not be used on the three questions in Part B.

Directions: Solve each problem in the provided space. Be efficient with the use of your time. A calculator may be used on Part A to solve equations, find derivatives at specific points, and evaluate definite integrals. A calculator may not be used on Part B.

Write neatly so that your work can be read. Cross out any mistakes. Work that is erased or crossed out will not be scored.

Before beginning work on Section II, you may want to look over all of the questions. Not everyone is expected to be able to complete every question.

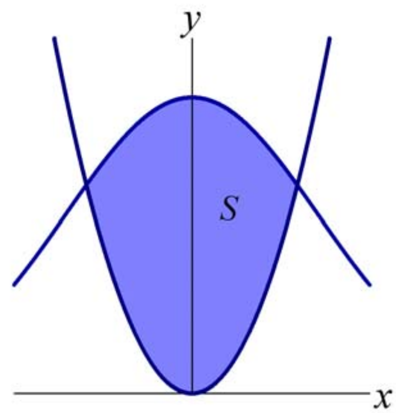
As you work this exam, be sure to do the following:

- Show your work. Label tables, functions, and graphs that you create. Your solutions will be scored based on the correctness of the problem solving process used in addition to the final answer. To receive full credit, you must provide supporting work. When justifying your answers, use mathematical reasons.
- Use standard mathematical notation instead of calculator syntax. For example, $\text{fnInt}\left(x^2-2x,x,3,7\right)$ may not be used to represent $\int_3^7\left(x^2-2x\right)dx$.
- Numeric and algebraic answers do not need to be simplified (unless otherwise specified). Decimal approximations in final answers should be accurate to three decimal places.
- The domain of each function f is the set of all real numbers for which $f(x)$ is defined. If the domain of a particular function differs from this, it will be specified in the problem.

Part A A calculator may be required to solve some problems.

Question 1

Let S be the region bounded by the graphs of $f(x) = e^{-x^2}$ and $g(x) = 2x^2$.



(a) Find the area of the region S.

(b) Find the volume of the solid generated when the region S is rotated about the x-axis.

(c) The region S is the base of a solid for which each cross section perpendicular to the x-axis is a square with diameter in the xy plane. Find the volume of the solid.

Question 2

Let $F(x)=\int_0^x \sin^2(t)dt$ on the interval $[0,\pi]$.

(a) Approximate $F(\pi)$ using the trapezoid rule with $n=4$.

(b) Find $F'(x)$.

(c) Find the average value of $F'(x)$ on the interval $[0,\pi]$.

Question 3

A particle moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 2t - 4$. At time $t = 1$, the velocity of the particle is $v(1) = -1$ and its position is $x(1) = \frac{4}{3}$.

(a) Write an expression for the velocity of the particle $v(t)$.

(b) At what value(s) of t does the particle change direction?

(c) Write an expression for the position $x(t)$ of the particle.

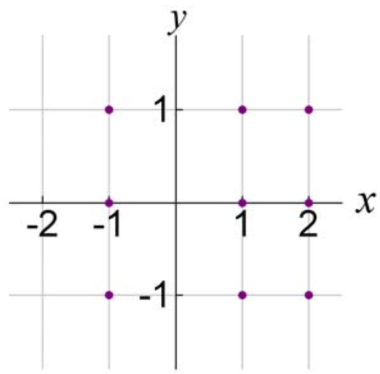
(d) Find the total distance traveled by the particle from $t = 1$ to $t = 4$.

Part B A calculator may not be used.

Question 4

Consider the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$.

(a) On the given axes, draw a slope field for the differential equation at the nine points shown.



(b) Find the particular solution $y = g(x)$ to the differential equation with initial condition $g(1) = -1$. Then state the domain of $g(x)$.

Question 5

A twice-differentiable function g has the following properties: $g(0)=3$, $g'(0)=5$, and $g''(0)=-1$.

(a) A function f is defined as $f(x)=3^{kx}+g(x)$ where k is constant. Find $f'(0)$ and $f''(0)$ in terms of k. Show how you obtained your answer.

(b) The function h is defined as $h(x)=\sin(ax)g(x)$ where a is constant. Find $h'(x)$ and determine the equation of the tangent line of the graph of h at $x=0$.

Question 6

t (minutes)	0	10	20	30	40	50	60	70	80
$v(t)$ (meters per minute)	0	180	172	160	158	164	166	152	170

The velocity of Runner A at selected times during a race are shown in the table above. At all times during the race, the runner's velocity was nonnegative. The runner had initial position 0 meters. (She was at the starting line.)

(a) Find the average acceleration of the Runner A from $t = 0$ minutes to $t = 80$ minutes. Label your answer with appropriate units.

(b) Explain the meaning of $\int_0^{80} v(t)\,dt$ as related to the runner, making sure to use appropriate units as a part of your explanation. Then use a midpoint Riemann sum with 4 subintervals of the same length to approximate $\int_0^{80} v(t)\,dt$.

(c) At time $t = 8$, Runner B leaves the starting line with a velocity of 160 meters per minute. Runner B accelerates by $a(t) = \dfrac{1}{\sqrt{t+1}}$ meters per minute per minute. Is Runner A or Runner B running faster at $t = 80$ minutes?